

Can a spectator scalar field enhance inflationary tensor mode?

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Abstract. We consider the possibility of enhancing the inflationary tensor mode by introducing a spectator scalar field with a small sound speed which induces gravitational waves as a second order effect. We analytically obtain the power spectra of gravitational waves and curvature perturbation induced by the spectator scalar field. We found that the small sound speed amplifies the curvature perturbation much more than the tensor mode and the current observational constraint forces the induced gravitational waves to be negligible compared with those from the vacuum fluctuation during inflation.

Keywords: inflation, primordial gravitational waves (theory)

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1 Introduction

Recently, cosmological primordial gravitational waves (GWs) attract much attention. In slow-roll inflation [1–3] (for the latest pedagogical review of inflation, see ref. [4]), it is known that the stochastic gravitational waves are generated from the vacuum fluctuation, and the power spectrum is proportional to the energy scale of inflation ρ_{inf} ,

$$\mathcal{P}_h^{\text{vac}} = \frac{2H^2}{\pi^2 M_{\text{Pl}}^2} \propto \rho_{\text{inf}}, \quad (1.1)$$

where H is the Hubble parameter during inflation and M_{Pl} is the reduced Planck mass [5]. Therefore if one measures $\mathcal{P}_h^{\text{vac}}$, ρ_{inf} can be observationally revealed.

However, it should be noted that observing the amplitude of primordial GWs does not necessarily mean that we can determine ρ_{inf} immediately. It is because not only $\mathcal{P}_h^{\text{vac}}$ but also GWs from other sources possibly contribute the observed GWs. Any theoretical argument or observational evidence guarantees that the observed GWs are dominated by $\mathcal{P}_h^{\text{vac}}$. Thus it is important to explore an alternative possibility of the generation of GWs in the primordial universe.

In general, GWs are sourced by the anisotropic component of stress energy tensor. Since a vector field naturally provides such an anisotropic stress, several mechanisms in which vector fields produced during inflation source GWs are investigated [6–11].¹ Nevertheless, the authors in ref. [12] pointed out that the spatial kinetic energy of a scalar field also sources GWs and it can be amplified when the sound speed of the scalar field is significantly smaller than unity.

In this paper, we consider a spectator scalar field with a generalized Lagrangian which gives a nontrivial sound speed. The GWs and the curvature perturbation induced by the scalar field are analytically calculated. We have found that the induced curvature perturbation becomes much larger than the induced GWs. The requirement that the induced curvature perturbation cannot exceed the observed value puts a constraint on the amplitude of the induced GWs. Consequently, it is shown that the GWs induced by a spectator field with a small sound speed is restricted to be much smaller than $\mathcal{P}_h^{\text{vac}}$. Finally, we extend

¹For other mechanisms, see also ref. [13].

the action of the additional scalar into a more generic form which contains the Galileon-like term. Even in this case, however, we found the induced GWs is strictly limited and cannot be dominant.

The rest of the paper is organized as follows. In Sec. 2, we perturb the action and obtain the power spectrum of the spectator field perturbation. In Sec. 3, the power spectra of the induced GWs and the curvature perturbation are derived and their constraints are discussed. In Sec. 4, we develop the understanding of the reason why such a stringent constraint on the induced GWs is obtained. In Sec. 5, the extended action of the spectator field is briefly argued. We conclude in Sec. 6.

2 Perturbed Action

We consider the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}M_{\text{Pl}}^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + P(X, \sigma), \quad (2.1)$$

where ϕ is the inflaton, $V(\phi)$ is its potential, σ is a spectator field, and $X \equiv \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$. In this paper, the inflaton ϕ is assumed to be responsible for both the occurrence of inflation and the generation of the scalar perturbations imprinted in the cosmic microwave background radiation (CMB) [14, 15]. On the other hand, the σ field is supposed to generate gravitational waves through its second order perturbations. For the moment, we assume that the Lagrangian of σ is an arbitrary function of X and σ , $P(X, \sigma)$, while we further extend it in Sec. 5. In this section, we perturb the action and derive the equations of motion for the perturbed fields.

In the (3+1) decomposition, the metric is given by

$$ds^2 = N^2 dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (2.2)$$

where we incorporate metric perturbations around the flat FLRW metric as,

$$N = 1 + \delta N, \quad N_i = \partial_i \psi, \quad \gamma_{ij} = a^2 \left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_j^k \right), \quad (2.3)$$

working in the flat gauge for the scalar perturbations and the transverse-traceless (T.T.) gauge for the tensor perturbations. One can show that the gravity sector of the perturbed action up to the second order is given by

$$\begin{aligned} S_g^{(1,2)} = \int dt d^3x a^3 & \left[3M_{\text{Pl}}^2 H^2 \delta N \quad (\text{1st order}) \right. \\ & \left. - 3M_{\text{Pl}}^2 H^2 (\delta N)^2 - 2M_{\text{Pl}}^2 H \delta N a^{-2} \partial_i^2 \psi + \frac{M_{\text{Pl}}^2}{8} \left(\dot{h}_{ij} \dot{h}_{ij} - a^{-2} \partial_k h_{ij} \partial_k h_{ij} \right) \quad (\text{2nd order}) \right]. \end{aligned} \quad (2.4)$$

Here we ignore the third and higher order terms in the gravity sector. Although they include $\mathcal{O}(h\delta\sigma^2)$ coupling terms, these terms are slow-roll suppressed compared to similar terms in the matter sector and hence they are sub-leading [16].

We now consider the matter sector of the action. The two scalar fields are decomposed into the background part and the perturbation,

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x}), \quad \sigma(t, \mathbf{x}) = \sigma_0(t) + \delta\sigma(t, \mathbf{x}). \quad (2.5)$$

The calculation of the perturbed matter action is straightforward. First, let us compute the perturbed Lagrangian of the σ field. The perturbation expansion of $X \equiv \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$ is given by

$$\begin{aligned}
X = & \frac{1}{2}\dot{\sigma}_0^2 \quad (0\text{th order}) + \dot{\sigma}_0\dot{\sigma} - \dot{\sigma}_0^2\delta N \quad (1\text{st order}) \\
& + \frac{1}{2}\dot{\sigma}^2 - \dot{\sigma}_0a^{-2}\partial_i\psi\partial_i\delta\sigma - 2\delta N\dot{\sigma}_0\dot{\sigma} + \frac{3}{2}\dot{\sigma}_0^2\delta N^2 - \frac{1}{2}a^{-2}(\partial_i\delta\sigma)^2 \quad (2\text{nd order}) \\
& - \dot{\sigma}a^{-2}\partial_i\psi\partial_i\delta\sigma - \delta N\left(\dot{\sigma}^2 - 2\dot{\sigma}_0a^{-2}\partial_i\psi\partial_i\delta\sigma\right) + 3\dot{\sigma}_0\dot{\sigma}\delta N^2 \\
& - 2\dot{\sigma}_0^2\delta N^3 + \frac{1}{2}a^{-2}h_{ij}\partial_i\delta\sigma\partial_j\delta\sigma \quad (3\text{rd order}) + \mathcal{O}(\delta\sigma^4). \tag{2.6}
\end{aligned}$$

Then one finds

$$\begin{aligned}
NP(X, \sigma) = & (1 + \delta N)P(X, \sigma) \\
= & P^{(0)} \quad (0\text{th order}) \\
& + P^{(0)}\delta N + P_X^{(0)}\left(\dot{\sigma}_0\dot{\sigma} - \dot{\sigma}_0^2\delta N\right) + P_\sigma^{(0)}\dot{\sigma} \quad (1\text{st order}) \\
& + \frac{1}{2}P_X^{(0)}\left[\dot{\sigma}^2 - 2\dot{\sigma}_0a^{-2}\partial_i\psi\partial_i\delta\sigma - 2\dot{\sigma}_0\dot{\sigma}\delta N + \dot{\sigma}_0^2(\delta N)^2 - a^{-2}(\partial_i\delta\sigma)^2\right] \\
& + \frac{1}{2}P_{XX}^{(0)}\left(\dot{\sigma}_0\dot{\sigma} - \dot{\sigma}_0^2\delta N\right)^2 + \frac{1}{2}P_{\sigma\sigma}^{(0)}(\dot{\sigma})^2 + P_\sigma^{(0)}\dot{\sigma}\delta N \quad (2\text{nd order}) \\
& + \frac{1}{2}P_{XX}^{(0)}\dot{\sigma}_0(\dot{\sigma})^3 - \left(\frac{1}{2}P_X^{(0)} - 2P_{XX}^{(0)}\dot{\sigma}_0\right)(\dot{\sigma})^2\delta N + \left(P_X^{(0)} + \frac{5}{2}P_{XX}^{(0)}\dot{\sigma}_0\right)\dot{\sigma}_0\dot{\sigma}\delta N^2 \\
& - \left(\frac{1}{2}P_X^{(0)} + P_{XX}^{(0)}\dot{\sigma}_0\right)\dot{\sigma}_0^2(\delta N)^3 + \left(P_X^{(0)} + P_{XX}^{(0)}\dot{\sigma}_0\right)\left(\dot{\sigma}_0\delta N - \dot{\sigma}\right)a^{-2}\partial_i\psi\partial_i\delta\sigma \\
& - \frac{1}{2}P_{XX}^{(0)}\dot{\sigma}_0\dot{\sigma}a^{-2}(\partial_i\delta\sigma)^2 - \frac{1}{2}\left(P_X^{(0)} - P_{XX}^{(0)}\dot{\sigma}_0\right)\delta Na^{-2}(\partial_i\delta\sigma)^2 \\
& + \frac{1}{2}P_{\sigma\sigma}^{(0)}(\dot{\sigma})^2\delta N + \frac{1}{2}P_X^{(0)}h_{ij}a^{-2}\partial_i\delta\sigma\partial_j\delta\sigma \quad (3\text{rd order}) + \mathcal{O}(\delta\sigma^4), \tag{2.7}
\end{aligned}$$

where $P_{X^n}^{(0)} \equiv \partial^n P / \partial X^n|_{X=\dot{\sigma}_0^2/2, \sigma=\sigma_0}$. Note that we suppress the terms in proportion to $P_{X\sigma}^{(0)}, P_{\sigma\sigma\sigma}^{(0)}$ and other higher derivatives which do not yield the $h\delta\sigma^2$ coupling. A general multi-field perturbed action can be found in ref. [17] while it does not include the tensor perturbations. One can easily obtain the perturbed lagrangian of the ϕ sector by making replacements, $\delta\sigma \rightarrow \delta\phi, P_X^{(0)} \rightarrow 1, P_{XX}^{(0)} \rightarrow 0, P_\sigma^{(0)} \rightarrow -V_\phi^{(0)}$ and $P^{(0)} \rightarrow \dot{\phi}_0^2/2 + V^{(0)}$ in eq. (2.7).

Now we have the perturbed action with the four scalar perturbation quantities, $\delta N, \psi, \delta\phi$ and $\delta\sigma$. However, the Hamiltonian and momentum constraints of the second order action eliminates the two of them,

$$2M_{\text{Pl}}^2 H \delta N = \dot{\phi}_0 \delta\phi + P_X^{(0)} \dot{\sigma}_0 \delta\sigma, \tag{2.8}$$

$$-2M_{\text{Pl}}^2 H a^{-2} \partial_i^2 \psi = \left(6M_{\text{Pl}}^2 H^2 - \dot{\phi}_0^2 - K \dot{\sigma}_0^2\right) \delta N + \dot{\phi}_0 \dot{\sigma} \delta\phi + V_\phi^{(0)} \delta\phi + K \dot{\sigma}_0 \dot{\sigma} - P_\sigma^{(0)} \dot{\sigma}, \tag{2.9}$$

with $K \equiv P_X + P_{XX}\dot{\sigma}_0^2$. Using these constraint equations and eliminating δN and ψ , we

obtain the second order action of $\delta\phi$ and $\delta\sigma$ as [17]

$$S^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[(\dot{\delta\phi})^2 + K(\dot{\delta\sigma})^2 - a^{-2}(\partial_i \delta\phi)^2 - P_X a^{-2}(\partial_i \delta\sigma)^2 - \mu_\phi^2 (\delta\phi)^2 - \mu_\sigma^2 (\delta\sigma)^2 - \Omega \delta\phi \delta\sigma - \tilde{\Omega} \delta\phi \dot{\delta\sigma} \right], \quad (2.10)$$

where

$$\mu_\phi^2 \equiv V_{\phi\phi} + \frac{3\dot{\phi}_0^2}{2M_{\text{Pl}}^2} + \frac{\dot{\phi}_0 V_\phi}{M_{\text{Pl}}^2 H} - \frac{\dot{\phi}_0^2}{4M_{\text{Pl}}^4 H^2} \left(\dot{\phi}_0^2 + K\dot{\sigma}_0^2 \right) - \frac{\partial_t(a^3 \dot{\phi}_0^2/H)}{2M_{\text{Pl}}^2 a^3}, \quad (2.11)$$

$$\mu_\sigma^2 \equiv -P_{\sigma\sigma} + \frac{3P_X^2 \dot{\sigma}_0^2}{M_{\text{Pl}}^2} - \frac{P_\sigma P_X \dot{\sigma}_0}{M_{\text{Pl}}^2 H} - \frac{P_X^2 \dot{\sigma}_0^2}{4M_{\text{Pl}}^4 H^2} \left(\dot{\phi}_0^2 + K\dot{\sigma}_0^2 \right) - \frac{\partial_t(a^3 K P_X \dot{\sigma}_0^2/H)}{2M_{\text{Pl}}^2 a^3}, \quad (2.12)$$

$$\Omega \equiv 3 \frac{\dot{\phi}_0 P_X \dot{\sigma}_0}{M_{\text{Pl}}^2} - \frac{P_\sigma \dot{\phi}_0}{M_{\text{Pl}}^2 H} + \frac{V_\phi P_X \dot{\sigma}_0}{M_{\text{Pl}}^2 H} - \frac{\dot{\phi}_0 P_X \dot{\sigma}_0}{2M_{\text{Pl}}^4 H^2} \left(\dot{\phi}_0^2 + K\dot{\sigma}_0^2 \right) - \frac{\partial_t(a^3 \dot{\phi}_0 P_X \dot{\sigma}_0/H)}{M_{\text{Pl}}^2 a^3}, \quad (2.13)$$

$$\tilde{\Omega} \equiv \frac{\dot{\phi}_0 K \dot{\sigma}_0}{M_{\text{Pl}}^2 H} + \frac{\dot{\phi}_0 P_{XX} \dot{\sigma}_0^3}{M_{\text{Pl}}^2 H}. \quad (2.14)$$

Note we omit the superscript “(0)” hereafter. To canonically normalize the fields, we redefine

$$\chi \equiv a\delta\phi, \quad \Sigma \equiv a\sqrt{K}\delta\sigma. \quad (2.15)$$

With these new variables, the second-order action reads

$$S^{(2)} = \frac{1}{2} \int d\eta d^3x \left[\chi'^2 - (\partial_i \chi)^2 + \left(\frac{a''}{a} - a^2 \mu_\phi^2 \right) \chi^2 + \Sigma'^2 - c_s^2 (\partial_i \Sigma)^2 + \left(\frac{(a\sqrt{K})''}{a\sqrt{K}} - a^2 \mu_\sigma^2 \right) \Sigma^2 + \frac{a}{\sqrt{K}} \left(\tilde{\Omega} \frac{(a\sqrt{K})'}{a\sqrt{K}} - a\Omega \right) \chi \Sigma - \frac{a}{\sqrt{K}} \tilde{\Omega} \chi \Sigma' \right], \quad (2.16)$$

where the prime denotes the derivative with respect to the conformal time η and we introduce the sound speed of the canonical field Σ ,

$$c_s^2 \equiv \frac{P_X}{K} = \frac{P_X}{P_X + P_{XX} \dot{\sigma}_0^2}. \quad (2.17)$$

The equations of motion of the two canonical fields are given by

$$\chi'' - \partial_i^2 \chi + \left(a^2 \mu_\phi^2 - \frac{a''}{a} \right) \chi = \frac{a}{\sqrt{K}} \left[\left(\tilde{\Omega} \frac{(a\sqrt{K})'}{a\sqrt{K}} - a\Omega \right) \Sigma - \tilde{\Omega} \Sigma' \right], \quad (2.18)$$

$$\Sigma'' - c_s^2 \partial_i^2 \Sigma + \left[a^2 \mu_\sigma^2 - \frac{(a\sqrt{K})''}{a\sqrt{K}} \right] \Sigma = \frac{a}{\sqrt{K}} \left(\tilde{\Omega} \frac{(a\sqrt{K})'}{a\sqrt{K}} - a\Omega \right) \chi + \left(\frac{a}{\sqrt{K}} \tilde{\Omega} \chi \right)'. \quad (2.19)$$

Since these equations are coupled to each other due to the mixing terms (see the third line in eq. (2.16)), it is hard to solve them if the mixing is significantly strong. Moreover, if the masses, μ_ϕ^2 and μ_σ^2 , are not much less than H^2 , their fluctuations are not generated

during inflation. Thus we explore the condition in which both the mixing and their mass are negligible and we focus on these cases in the following section.

The inflaton mass, μ_ϕ^2 , is evaluated as

$$\frac{\mu_\phi^2}{H^2} \simeq 3\eta_\phi - 6\epsilon_H + 6\sqrt{\epsilon_\phi\epsilon_H} - P_X \frac{\epsilon_H^2 \dot{\sigma}_0^2}{c_s^2 \dot{\phi}_0^2} + \mathcal{O}(\epsilon^2), \quad (2.20)$$

where $\epsilon_H \equiv -\dot{H}/H^2$, $\epsilon_\phi \equiv M_{\text{Pl}}^2 V_\phi^2 / 2V^2$, $\eta_\phi \equiv M_{\text{Pl}}^2 V_{\phi\phi} / V$ as usual. We also use the background equation, $-2M_{\text{Pl}}^2 \dot{H} = \dot{\phi}_0^2 + P_X \dot{\sigma}_0^2$. In eq. (2.20), only the last term can be large for a very small c_s . It requires a condition,

$$c_s^2 \gg \left| \epsilon_H^2 \frac{P_X \dot{\sigma}_0^2}{\dot{\phi}_0^2} \right|, \quad (2.21)$$

for the inflaton mass to be negligibly small. Provided $P_{\sigma\sigma} \lesssim V_{\phi\phi}$, $P_\sigma \lesssim V_\phi$ and $P_X \dot{\sigma}_0 \lesssim \dot{\phi}_0$ which are natural conditions for a spectator field, one can show that eq. (2.21) guarantees $\mu_\sigma^2 \ll H^2$ and $\Omega \ll H^2$. However, for a small c_s , one finds

$$\frac{\tilde{\Omega}}{H} \simeq 4 \frac{\epsilon_H}{c_s^2} \frac{P_X \dot{\sigma}_0}{\dot{\phi}_0}. \quad (2.22)$$

To ignore the mixing, we need an additional condition;

$$c_s^2 \gg \left| \epsilon_H \frac{P_X \dot{\sigma}_0}{\dot{\phi}_0} \right|. \quad (2.23)$$

and they do not appear if σ has a usual kinetic term.

When the two conditions, eqs. (2.21) and (2.23), are satisfied and the slow-roll parameters are sufficiently small, the mass terms and the mixing terms are safely ignored. Then we obtain the mode functions of the two fields as

$$\chi_k \simeq \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right), \quad \Sigma_k \simeq \frac{e^{-ic_s k \eta}}{\sqrt{2c_s k}} \left(1 - \frac{i}{c_s k \eta} \right), \quad (2.24)$$

where the time variation of K is assumed to be negligible compared with a . The power spectrum of the original fields on super-horizon scales are given by

$$\mathcal{P}_{\delta\phi} \simeq \frac{H^2}{4\pi^2}, \quad \mathcal{P}_{\delta\sigma} \simeq \frac{1}{c_s^3 K} \frac{H^2}{4\pi^2} = \frac{1}{c_s P_X} \frac{H^2}{4\pi^2}. \quad (2.25)$$

Note that the power spectrum of the σ field is amplified by the factor of $(c_s P_X)^{-1}$.

3 Induced curvature and graviton perturbations

In this section, we calculate the curvature perturbations and gravitational waves induced by the σ field through the third-order terms in the perturbed action. The third-order action contains many terms,

$$\begin{aligned} S^{(3)} \supset \int dt d^3x a^3 \Big[& \frac{1}{2} P_X h_{ij} a^{-2} \partial_i \delta\sigma \partial_j \delta\sigma - \frac{1}{2} (P_X - P_{XX} \dot{\sigma}_0^2) a^{-2} (\partial_i \delta\sigma)^2 \delta N \\ & - \left(\frac{1}{2} P_X - 2P_{XX} \dot{\sigma}_0^2 \right) (\dot{\delta\sigma})^2 \delta N + \frac{1}{2} P_{\sigma\sigma} (\delta\sigma)^2 \delta N + \dots \Big], \end{aligned} \quad (3.1)$$

where we have shown only a few terms. Remember δN can be written by $\delta\phi$ and $\delta\sigma$ using eq. (2.8). Actually, there is only one $h(\delta\sigma)^2$ coupling term (the first term in eq. (3.1)), except for the slow-roll suppressed terms in the gravity sector. However, there are many $\delta\phi(\delta\sigma)^2$ coupling terms and it is not transparent which one is most significant. Then we focus on the term with $\delta N(\partial_i\delta\sigma)^2$ (the second term in eq. (3.1)) because it has a similar form to the graviton coupling term and it is easy to compare them. As we see later, the curvature perturbation induced only by this term excludes the dominant production of gravitational waves via the spectator field. Thus this treatment is conservative and sufficient.

Since σ is a spectator field, the comoving curvature perturbation is determined by the inflaton as

$$\mathcal{R} \simeq -\frac{H}{\dot{\phi}_0}\delta\phi \simeq -\frac{2M_{\text{Pl}}H^2}{\dot{\phi}_0^2}\delta N \simeq -\frac{\delta N}{\epsilon_H}. \quad (3.2)$$

As we see later, to produce the induced gravitons significantly, c_s should be much smaller than unity. Thus one can approximate

$$c_s^2 = \frac{P_X}{P_X + P_{XX}\dot{\sigma}_0^2} \ll 1 \quad \implies \quad K \equiv P_X + P_{XX}\dot{\sigma}_0^2 \simeq P_{XX}\dot{\sigma}_0^2. \quad (3.3)$$

Then the first line in eq. (3.1) reads

$$S_{\text{calc}}^{(3)} = \int d\eta d^3x a^2 \left[\frac{1}{2} P_X h_{ij} \partial_i \delta\sigma \partial_j \delta\sigma - \frac{1}{2} \epsilon_H K \mathcal{R} \partial_i \delta\sigma \partial_i \delta\sigma \right]. \quad (3.4)$$

On the other hand, substituting eq. (3.2) into eq. (2.10), we obtain the relevant second order action as

$$S_{\mathcal{R},h}^{(2)} = \int d\eta d^3x \left[a^2 \epsilon M_{\text{Pl}}^2 (\mathcal{R}'^2 - (\partial_i \mathcal{R})^2) + \frac{a^2 M_{\text{Pl}}^2}{8} (h'_{ij} h'_{ij} - \partial_k h_{ij} \partial_k h_{ij}) \right]. \quad (3.5)$$

Note that all sub-leading terms are dropped and h_{ij} terms come from eq. (2.4). Combining it with eq. (3.4), one obtains the equations of motion as

$$\mathcal{R}'' + 2\mathcal{H}\mathcal{R}' - \partial_i^2 \mathcal{R} = -\frac{K}{4M_{\text{Pl}}^2} \partial_i \delta\sigma \partial_i \delta\sigma, \quad (3.6)$$

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \partial_k^2 h_{ij} = \frac{2P_X}{M_{\text{Pl}}^2} \tilde{T}_{ij}^{lm} \partial_l \delta\sigma \partial_m \delta\sigma. \quad (3.7)$$

Here \tilde{T}_{ij}^{lm} is the projection tensor into the T.T. component defined by

$$\tilde{T}_{ij}^{lm}(\mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[e_{ij}^+(\mathbf{k}) e_{lm}^+(\mathbf{k}) + e_{ij}^-(\mathbf{k}) e_{lm}^-(\mathbf{k}) \right]. \quad (3.8)$$

Here e_{ij}^\pm are the polarization tensors which are written in terms of the polarization vectors $e_i(\mathbf{k})$ and $\bar{e}_i(\mathbf{k})$ as

$$e_{ij}^+(\mathbf{k}) = \frac{1}{\sqrt{2}} [e_i(\mathbf{k}) e_j(\mathbf{k}) - \bar{e}_i(\mathbf{k}) \bar{e}_j(\mathbf{k})], \quad e_{ij}^-(\mathbf{k}) = \frac{1}{\sqrt{2}} [e_i(\mathbf{k}) \bar{e}_j(\mathbf{k}) + \bar{e}_i(\mathbf{k}) e_j(\mathbf{k})], \quad (3.9)$$

where $e_i(\mathbf{k})$ and $\bar{e}_i(\mathbf{k})$ are two basis vectors which are orthogonal to each other and \mathbf{k} . The only differences between the source terms of \mathcal{R} and h_{ij} are the coefficients and the projection

tensor. In what follows, we focus on the calculation of h_{ij} . One can solve the equation of \mathcal{R} in a similar manner.

Equation (3.7) can be solved by the Green function method. The Green function $g_k(\eta, \tau)$ which satisfies

$$g_k'' + 2\mathcal{H}g_k' + k^2g_k = \delta(\eta - \tau), \quad (3.10)$$

is given by

$$g_k(\eta, \tau) = \frac{\theta(\eta - \tau)}{k^3\tau^2} \Re e \left[e^{ik(\eta - \tau)} (1 - ik\eta)(-i + k\tau) \right], \quad (3.11)$$

where $\theta(\eta)$ is the step function and $\Re e[\dots]$ represents the real part of $[\dots]$. Using this Green function, one finds the inhomogeneous solutions of eq. (3.7) as

$$h_{\mathbf{k}}^{\pm}(\eta) = \frac{2P_X}{M_{\text{Pl}}^2} \int \frac{d^3p d^3q}{(2\pi)^3} \delta(\mathbf{p} + \mathbf{q} - \mathbf{k}) e_{ij}^{\pm}(\mathbf{k}) p_i q_j \int_{-\infty}^{\infty} d\tau g_k(\eta, \tau) \sigma_{\mathbf{p}}(\tau) \sigma_{\mathbf{q}}(\tau). \quad (3.12)$$

Substituting them into the definitions of the power spectrum,

$$\langle h_{\mathbf{k}}^{\pm}(\eta) h_{\mathbf{k}'}^{\pm}(\eta) \rangle = \frac{2\pi^2}{k^3} \delta(\mathbf{k} + \mathbf{k}') \mathcal{P}_h^{\pm}(k, \eta), \quad (3.13)$$

one obtains

$$\begin{aligned} \frac{2\pi^2}{k^3} \delta(\mathbf{k} + \mathbf{k}') \mathcal{P}_h^{\pm}(k, \eta) = & \frac{4P_X^2}{M_{\text{Pl}}^4} \int \frac{d^3p d^3q d^3p' d^3q'}{(2\pi)^6} \delta(\mathbf{p} + \mathbf{q} - \mathbf{k}) \delta(\mathbf{p}' + \mathbf{q}' - \mathbf{k}') e_{ij}^{\pm}(\mathbf{k}) e_{ml}^{\pm}(\mathbf{k}') p_i q_j p'_m q'_l \\ & \times \int_{-\infty}^{\infty} d\tau d\tau' g_k(\eta, \tau) g_{k'}(\eta, \tau') \langle \sigma_{\mathbf{p}}(\tau) \sigma_{\mathbf{q}}(\tau) \sigma_{\mathbf{p}'}(\tau') \sigma_{\mathbf{q}'}(\tau') \rangle. \end{aligned} \quad (3.14)$$

Since we are treating the source terms of \mathcal{R} and h_{ij} due to the spectator field σ in eqs. (3.6) and (3.7) as classical stochastic quantities, momentum integrations in eqs. (3.12) and (3.14) are performed only in the domain where the quantum operator $\sigma_{\mathbf{p}}$ behaves as a classical stochastic variable. Specifically we introduce a parameter γ smaller than unity such that one can approximate

$$\sigma_{\mathbf{p}}(\eta) \cong \frac{H}{\sqrt{2c_s P_X} p^{3/2}} \left(\hat{a}_{\mathbf{p}} + \hat{a}_{-\mathbf{p}}^{\dagger} \right), \quad (3.15)$$

for $|c_s p \eta| < \gamma$, where $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^{\dagger}$ are creation and annihilation operators which satisfy the usual commutation relation, $[\hat{a}_{\mathbf{k}}, \hat{a}_{-\mathbf{k}'}^{\dagger}] = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$. Then both $\sigma_{\mathbf{p}}(\eta)$ and its canonically conjugate momentum variable have the same operator dependence proportional to $\hat{a}_{\mathbf{p}} + \hat{a}_{-\mathbf{p}}^{\dagger}$ and commute with each other.

Thus replacing $\sigma_{\mathbf{p}}$ in eq. (3.14) by

$$\sigma_{\mathbf{p}}(\eta) \cong \frac{H}{\sqrt{2c_s P_X} p^{3/2}} \theta(\gamma + c_s p \eta) \left(\hat{a}_{\mathbf{p}} + \hat{a}_{-\mathbf{p}}^{\dagger} \right), \quad (3.16)$$

one finds

$$\begin{aligned} & \langle \sigma_{\mathbf{p}}(\tau) \sigma_{\mathbf{q}}(\tau) \sigma_{\mathbf{p}'}(\tau') \sigma_{\mathbf{q}'}(\tau') \rangle \\ &= \frac{H^4}{4P_X^2 c_s^2} (p q p' q')^{-\frac{3}{2}} \theta(\gamma + c_s p \tau) \theta(\gamma + c_s q \tau) \theta(\gamma + c_s p' \tau') \theta(\gamma + c_s q' \tau') \\ & \quad \times (2\pi)^6 [\delta(\mathbf{p} + \mathbf{q}') \delta(\mathbf{q} + \mathbf{p}') + \delta(\mathbf{p} + \mathbf{p}') \delta(\mathbf{q} + \mathbf{q}')]. \end{aligned} \quad (3.17)$$

Substituting it into eq. (3.14), and using the symmetry $\mathbf{p}' \leftrightarrow \mathbf{q}'$, we obtain

$$\mathcal{P}_h^\pm(\eta, k) = \pm \frac{k^3}{\pi^2 c_s^2} \frac{H^4}{M_{\text{Pl}}^4} \int d^3 p d^3 p' \delta(\mathbf{p} - \mathbf{p}' - \mathbf{k}) e_{ij}^\pm(\mathbf{k}) e_{ml}^\pm(\mathbf{k}) \frac{p_i p'_j p'_m p_l}{(pp')^3} \times \left[\int_{-\infty}^{\infty} d\tau g_k(\eta, \tau) \theta(\gamma + c_s p \tau) \theta(\gamma + c_s p' \tau) \right]^2, \quad (3.18)$$

where the property of the linear tensor polarization, $e_{ij}^\pm(-\mathbf{k}) = \pm e_{ij}^\pm(\mathbf{k})$, is used. The time integration can be analytically performed as

$$k^2 \int_{-\infty}^{\infty} d\tau g_k(\eta, \tau) \theta(\gamma + c_s p \tau) \theta(\gamma + c_s p' \tau) = 1 + \frac{\sin[k(\eta - \tilde{\eta}_p)]}{k \tilde{\eta}_p} - \frac{\eta}{\tilde{\eta}_p} \cos[k(\eta - \tilde{\eta}_p)], \quad (3.19)$$

with

$$\tilde{\eta}_p \equiv -\frac{\gamma}{c_s \max[p, p']}, \quad (3.20)$$

which is the sound horizon crossing time of either p or p' mode, whichever exits the horizon later. Finally, in the p integration, one can show that the contribution from $p \sim \gamma k / c_s$ is dominant. Then eq. (3.19) can be approximated by $1 - x \sin(x^{-1})$ for $c_s \ll \gamma$, where $x \equiv c_s p / \gamma k$. After the angular integration, one finds

$$\mathcal{P}_h^\pm(\eta, k) \simeq \pm \frac{8\gamma}{15\pi c_s^3} \frac{H^4}{M_{\text{Pl}}^4} \int_\epsilon^\xi dx [1 - x \sin(x^{-1})]^2, \quad (3.21)$$

where $\xi \gg 1$ and $\epsilon \ll 1$ are introduced to define the integration interval. Although the x integration cannot be performed analytically, a numerical calculation shows that it converges to $\approx 1/2$ for a sufficiently large ξ and small ϵ . Remembering $\mathcal{P}_h = \mathcal{P}_h^+ - \mathcal{P}_h^-$, one obtains

$$\mathcal{P}_h^{(\sigma)}(\eta, k) \simeq \frac{8\gamma}{15\pi c_s^3} \frac{H^4}{M_{\text{Pl}}^4}, \quad (3.22)$$

where the superscript “ (σ) ” denotes that this \mathcal{P}_h is induced by the σ field. In the same way as $\mathcal{P}_h^{(\sigma)}$, one can show the induced power spectrum of the curvature perturbation is given by

$$\mathcal{P}_{\mathcal{R}}^{(\sigma)}(\eta, k) \simeq \frac{\gamma}{32\pi c_s^7} \frac{H^4}{M_{\text{Pl}}^4}. \quad (3.23)$$

Thus a spectator field which induces the gravitational waves of eq. (3.22) inevitably produces the curvature perturbation of eq. (3.23) as well.

Since $H \ll M_{\text{Pl}}$, the induced \mathcal{P}_h , eq. (3.22), is negligible compared to the one coming from the vacuum fluctuation, eq. (1.1), unless the sound speed c_s takes a tiny value satisfying $c_s^3 < 4\pi\gamma H^2 / 15M_{\text{Pl}}^2$. In that case, however, the tensor-to-scalar ratio induced by the spectator field,

$$r_\sigma \equiv \frac{\mathcal{P}_h^{(\sigma)}}{\mathcal{P}_{\mathcal{R}}^{(\sigma)}} \simeq \frac{256}{15} c_s^4, \quad (3.24)$$

becomes very small. As a result, the requirement that the induced curvature perturbation must not exceed the observed value, $\mathcal{P}_{\mathcal{R}}^{(\sigma)} \leq \mathcal{P}_{\mathcal{R}}^{\text{obs}} \approx 2.2 \times 10^{-9}$, puts a lower bound on c_s and consequently constrains $\mathcal{P}_h^{(\sigma)}$ as

$$\frac{\mathcal{P}_h^{(\sigma)}}{\mathcal{P}_h^{\text{vac}}} \leq 2 \times 10^{-5} \gamma^{\frac{4}{7}} \left(\frac{H}{10^{14} \text{GeV}} \right)^{\frac{2}{7}}. \quad (3.25)$$

As mentioned above, we expect $\gamma \lesssim 1$ and it is known $H \lesssim 10^{14} \text{GeV}$ from the CMB observations [14, 15, 18]. Therefore the induced GW cannot dominate the GW from the vacuum fluctuation.

4 Interpretation

In the previous section, it was shown that the spectator field with a tiny sound speed produces the curvature perturbation $\mathcal{P}_{\mathcal{R}}^{(\sigma)} \propto c_s^{-7}$ larger than the gravitational waves, $\mathcal{P}_h^{(\sigma)} \propto c_s^{-3}$. This contrast originates in the difference of coupling constants. One can see in the right hand side of eqs. (3.6) and (3.7) that the ratio of the coupling constants is given by

$$\left| \frac{h\delta\sigma^2 \text{ coupling}}{\mathcal{R}\delta\sigma^2 \text{ coupling}} \right| \simeq \frac{8P_X}{K} = 8c_s^2. \quad (4.1)$$

Hence the $h(\delta\sigma)^2$ coupling is highly suppressed compared to the $\mathcal{R}(\delta\sigma)^2$ coupling for $c_s \ll 1$. Now let us take a closer look at what makes these two couplings so different.

The difference stems from the perturbative expansion of the action, $P(X, \sigma) = P + P_X \delta X + \frac{1}{2} P_{XX} (\delta X)^2 + \dots$. The $h(\delta\sigma)^2$ coupling appears in the perturbation of X , (see eq. (2.6))

$$\delta X \supset \frac{1}{2} a^{-2} h_{ij} \partial_i \delta\sigma \partial_j \delta\sigma. \quad (4.2)$$

Since this is already the third order, no other perturbation can be multiplied to this term. Thus only $P_X \delta X$ carries the $h\delta\sigma^2$ coupling. On the other hand, δX also has the following terms:

$$\delta X \supset \dot{\sigma}_0^2 \delta N - \frac{1}{2} a^{-2} (\partial_i \delta\sigma)^2, \quad (4.3)$$

where the first term in the right hand side is the first order of perturbation and includes $\delta\phi$ (or \mathcal{R}), while the second term is the second order. This time, $P_{XX} (\delta X)^2$ can carry the $\mathcal{R}(\delta\sigma)^2$ coupling terms. Therefore although the coefficient of the $h(\delta\sigma)^2$ coupling is only P_X , the $\mathcal{R}(\delta\sigma)^2$ coupling has $P_{XX} \dot{\sigma}_0^2$ in its coefficient. Meanwhile, since the sound speed is given by

$$c_s^2 = \frac{P_X}{P_X + P_{XX} \dot{\sigma}_0^2}, \quad (4.4)$$

$P_{XX} \dot{\sigma}_0^2 \gg P_X$ is necessary to make c_s tiny to boost $\mathcal{P}_h^{(\sigma)}$. However, it results in suppression of the P_X terms in comparison to the $P_{XX} \dot{\sigma}_0^2$ terms. Thus the $h(\delta\sigma)^2$ coupling is suppressed compared to the $\mathcal{R}(\delta\sigma)^2$ coupling.

This feature can also be understood as follows. A small sound speed means that the coefficient of the spacial kinetic term is smaller than that of the time kinetic term. Nevertheless, gravitational wave is induced by the spacial kinetic term of the σ field since the quadrupole component in the energy momentum tensor of a scalar field is given only by its

spacial kinetic energy. On the other hand, the adiabatic perturbation is sensitive to both the time and spacial kinetic energy. Therefore the suppression of the GW production in comparison with the curvature perturbation is a generic consequence of a small sound speed of a scalar field.

In summary, if the sound speed of the spectator field is much smaller than unity, its perturbation is amplified. As a result, both the gravitational waves and the curvature perturbation induced by its second order perturbation are boosted. However, the sound speed also controls the coupling constants of the $h(\delta\sigma)^2$ and $\mathcal{R}(\delta\sigma)^2$ coupling terms (see eq. (4.1)). As c_s becomes smaller, the $h(\delta\sigma)^2$ coupling is more suppressed compared to the $\mathcal{R}(\delta\sigma)^2$ coupling. Therefore, since a spectator field with a small sound speed induces the curvature perturbation much more than the gravitational waves, it cannot produce the dominant GW in a way that is consistent with the CMB observation.

5 Extension to the Galileon theory

So far the Lagrangian of the spectator field has been assumed to take a function of σ and X only. In this section, we show that the result obtained in the previous section does not change even if the action is extended to a more general form. Specifically we consider the Galileon-like theory [19–22],

$$\mathcal{L}_\sigma = P(X, \sigma) - G(X, \sigma)\Box\sigma, \quad (5.1)$$

where $G(X, \sigma)$ is an arbitrary function of X and σ and the other part of action is same as eq. (2.1). With this action, the sound speed of $\delta\sigma$ is given by [23]

$$c_s^2 = \frac{P_X + 2G_X(\ddot{\sigma}_0 + 2H\dot{\sigma}_0) - 2G_\sigma + G_{X\sigma}\dot{\sigma}_0^2 + G_{XX}\dot{\sigma}_0^2\ddot{\sigma}_0}{P_X + 6HG_X\dot{\sigma}_0 - 2G_\sigma - G_{X\sigma}\dot{\sigma}_0^2 + P_{XX}\dot{\sigma}_0^2 + 3HG_{XX}\dot{\sigma}_0^3}. \quad (5.2)$$

To make the sound speed small, terms proportional to P_{XX} or G_{XX} in the denominator have to be much larger than the other terms. As we discuss in the previous section, however, that leads to the suppression of the $h(\delta\sigma)^2$ coupling compared to the $\mathcal{R}(\delta\sigma)^2$ coupling because the $h(\delta\sigma)^2$ coupling does not include P_{XX} nor G_{XX} in the coupling constant while the $\mathcal{R}(\delta\sigma)^2$ coupling does. Indeed, the Galileon term additionally carries the following coupling terms:

$$-NG(X, \sigma)\Box\sigma \supset a^{-2} \left(G_\sigma - \frac{3}{2}H\dot{\sigma}_0 G_X \right) h_{ij}\partial_i\delta\sigma\partial_j\delta\sigma + \frac{3}{2}a^{-2}G_{XX}H\dot{\sigma}_0^3\delta N(\partial_i\delta\sigma)^2, \quad (5.3)$$

where we show only the leading terms. It is obvious that the discussion in Sec. 4 holds even in this Galileon case. Therefore we conclude that a spectator field with a small sound speed cannot produce the dominant GW even if its action is extended to the Galileon theory. This result implies that it is impossible for a *single scalar field with a small sound speed* to generate GW which is larger than the GW comes from the vacuum fluctuation.

6 Conclusion

It is important to explore an alternative source of primordial GWs other than GWs from the vacuum fluctuation because it can contribute the observed GWs and change the consequence on the inflation mechanism. We consider a spectator scalar field with the a generalized kinetic function and/or the Galileon-like action which gives it a small sound speed. The scalar field

induces both GWs and curvature perturbation which are analytically obtained as eq. (3.22) and eq. (3.23), respectively. Since a small sound speed makes the $\mathcal{R}(\delta\sigma)^2$ coupling much stronger than the $h(\delta\sigma)^2$ coupling, the induced curvature perturbation is considerably larger than the induced GWs. Then the CMB observation put the lower limit on the sound speed, and the stringent constraint on the induced GWs is derived. Consequently, we conclude that the GWs induced by the spectator scalar field cannot exceed $\mathcal{P}_h^{\text{vac}}$.

Note added

In the final stage of writing this manuscript a new paper by Biagetti et al [24] discussing the same topic showed up in the arXiv. Their conclusion is basically consistent with ours.

Acknowledgments

We would like to thank Takahiro Hayashinaka for useful comments. This work was supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. We acknowledge the support by Grant-in-Aid for JSPS Fellows No.248160 [T.F.], No. 242775 [S.Y.] and for Scientific Research (B) No. 23340058 [J.Y.].

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